INTEGRABLE SYSTEMS
A CELEBRATION OF EMMA PREVIATO'S 65TH BIRTHDAY

RON DONAGI AND TONY SHASKA

1. A word from the editors

These two volumes celebrate and honor Emma Previato, on the occasion of her 65th birthday. The present volume consists of 16 articles on and around the subject of integrable systems, one of the two main areas where Emma Previato has made many major contributions. The companion volume focuses on Emma's other major research area, algebraic geometry. The articles were contributed by Emma's coauthors, colleagues, students, and other researchers who have been influenced by Emma's work over the years. They present a very attractive mix of expository articles, historical surveys and cutting edge research.

Emma Previato is a mathematical pioneer, working in her two chosen areas, algebraic geometry and integrable systems. She has been among the first women to do research, in both areas. And her work in both areas has been deep and influential. Emma received a Bachelor's degree from the University of Padua in Italy, and a PhD from Harvard University under the direction of David Mumford in 1983. Her thesis was on hyperelliptic curves and solitons. The work on hyperelliptic curves has evolved and expanded into Emma's life-long interest in algebraic geometry. The work on solitons has led to her ongoing research on integrable systems, which is the subject of the present volume. Emma Previato has been a faculty member at the Department of Mathematics at Boston University since 1983. She has published nearly a hundred research articles, edited six books, and directed seven Ph.D dissertations. Her broader impact extends through her renowned teaching and her extensive mentoring activities. She runs AFRAMATH, an annual outreach symposium, and works tirelessly on several on- and o-campus mentoring programs. She has also founded and has been leading the activities of the Boston University chapters of MAA and AWM. She serves on numerous advisory boards.

The subject of integrable systems has a long and rich history. It brings together ideas and techniques from analysis, geometry, algebra and several branches of physics. Emma Previato's work has made important contributions in all of these directions. We review some of her accomplishments in section 2 below.

We tried to collect in this volume a broad range of articles covering most of these areas. We summarize these contributions in section 3.

The editors and many of the authors have enjoyed years of fruitful interactions with Emma Previato. We all join in wishing her many more years of health, productivity, and great mathematics.

Ron Donagi and Tony Shaska

2. Emma Previato's contributions

Emma Previato works in different areas, using methods from algebra, algebraic geometry, mechanics, differential geometry, analysis, and differential equations. The bulk of her research belongs to integrable equations. She is noted for often finding unexpected connections between integrability and many other areas, often including various branches of algebraic geometry.

2.1. **Early Activity.** As an undergraduate at the University of Padua, Italy, Emma wrote a dissertation on group lattices, followed by six journal publications [1–6]. With methods from algebra, initiated by Dedekind in the 19th century, this areas goal is to relate the group structure to the lattice of subgroups, and provide classifications for certain properties: an excellent overview is the article by Freese [7], a review of the definitive treatise by R. Schmidt, where results from all of Emma's papers are used to give one example, a lattice criterion for a finitely-generated group to be solvable.

2.2. **Ph.D. thesis and main area.** Emma's thesis [8], submitted at Harvard in 1983 under the supervision of D.B. Mumford [9], is still her most cited paper. Her thesis advisor was among the pioneers of this beautiful area, integrable equations, which grew and unified disparate parts of mathematics over the next twenty years, and is still very active. Emma's original tool for producing exact solutions to large classes of nonlinear PDEs, the Riemann theta function, remained one of her main interests.

2.3. **Theta Functions.** She later pursued more theoretical aspects of special functions, such as Prym theta functions [10–14] also surprisingly related to numerical results in conformal field theory, the Schottky problem [15], and Thetanulls [16].

2.4. **Algebraically Completely Integrable Systems.** The area of integrable PDEs is surprisingly related to algebraically completely integrable Hamiltonian systems, or ACIS, in the sense that algebro-geometric (aka finite- gap) solutions of integrable hierarchies linearize on Abelian varieties, which can be organized into angle variables for an ACIS over a suitable base, typically a subset of the moduli space of curves whose Jacobian is the fiber [11, 17]. Thanks to this discovery, the area integrates with classical geometric invariant theory, surface theory, and other traditional studies of algebraic geometry. With the appearance of the moduli spaces of vector bundles and Higgs bundles over a curve, at the hands of N. Hitchin in the 1980s, large families of ACIS were added to the examples, as well as theoretical algebro-geometric techniques. In [10, 18–20], Emma took up the challenge of generalizing the connection between ACIS and integrable hierarchies to curves beyond hyperelliptic. In [21], the families of curves are organized as divisors in surfaces.

2.5. **Higher rank and Higher-dimensional Spectra.** On the PDE side, the challenges were of two types. When the ring of functionson the (affine) spectral curve can be interpreted as differential operators with a higher dimensional space of common eigenfunctions, the fiber of the integrable system is no longer a Jacobian: it degenerates to a moduli space of higher-rank vector bundles, possibly with some auxiliary structures [22]. Neither the PDEs nor the integrable systems have been made explicit in higher rank in general. Some cases, however, are worked out in [23–27]. The other challenge is to increase the dimension of the spectral variety, for example from curve to surface. Despite much work, this problem too has arguably no explicit solution in general. An attempt to set up a general theory over a multi-dimensional version of the formal Universal Grassmann Manifold of Sato which hosts all linear flows of solutions of integrable hierarchies, is given in [28], and more concrete special settings are mentioned below, under the heading of Differential Algebra.

2.6. **Special Solutions.** Coverings of curves An important aspect of theta functions is their reducibility, a property whose investigation goes back to Weierstrass and his student S. Kowalevski. Given their special role in integrability, reducible theta functions are invaluable for applied mathematicians to approximate solutions, or even derive exact expressions and periods in terms of elliptic functions. To the algebro-geometric theory of Elliptic Solitons, initiated by I.M. Krichever and developed by A. Treibight and his thesis supervisor J.-L. Verdier, Emma contributed [12, 29–35], while [36, 37] generalize the reduction to hyperelliptic curves or Abelian subvarieties. More general aspects of elliptic (sub)covers are taken up in [38].

Another type of special solution is the one obtained by self-similarity [39]; the challenge here is to find an explicit relationship between the PDE flows and the deformation in moduli that obeys Painlev e-type equations: this is one reason why Emma's work has turned to a special function which is associated to Riemanns theta function but only exists on Jacobians: the sigma function (cf. the eponymous section below).

2.7. **Generalizing ACIS: Poncelet and Billiards:** Classical theorems of projective geometry can be generalized to ACIS [40, 41], while the challenge of matching them with integrable hierarchies is still ongoing [42].

2.8. **Generalizing ACIS: Hitchin Systems:** Explicit Hamiltonians for the Hitchin system are only available in theory: they are given explicit algebraic expression in [43] (cf. also [44], which led to work on the geometry of the moduli space of bundles [45]). An explicit integration in terms of special functions leads to the problem of non-commutative theta functions [46].

2.9. **Differential Algebra.** Differential Algebra is younger than Algebraic Geometry, but it has many features in common. Mumford gives credit to J.L. Burchnall and T.W. Chaundy for the first spectral curve, the Spectrum of a commutative ring of differential operators [47]. This is arguably the reason behind algebro-geometric solutions to integrable hierarchies. On the differential-algebra setting, Emma published [48, 49], connecting geometric properties of the curve with differential resultants, a major topic of elimination theory which is currently being worked out [50, 51] and naturally leads to the higher-rank solutions: their Grassmannian aspects are taken up in [52–56] the higher-dimensional spectral varieties are addressed in [57]. Other aspects of differential algebra are connected to integrability in [58] (the action of an Abelian vector field on the meromorphic functions of an Abelian variety) and [59] (a p-adic analog); in [60], the deformations act on modular forms.

2.10. **The Sigma Function.** Klein extended the definition of the (genus-one) Weierstrass sigma function to hyperelliptic curves and curves of genus three. H.F. Baker developed an in-depth theory of PDEs satisfied by the hyperelliptic sigma function, which plays a key role in recent work on integrable hierarchies (KdV- type, e.g.). Beginning in the 1990s, this theory of Kleinian sigma functions was revisited, originally by V.M. Buchstaber, V.Z. Enolskii and D.V. Leykin, much extended in scope, eventually to be developed for telescopic curves (a condition on the Weierstrass semigroup at a point). We go beyond the telescopic case in [61, 62], while we investigate the higher-genus analog of classical theorems in [63–70] and their connections with integrability in [71] and [72], which gives the first algebro-geometric solutions to a dispersionless integrable hierarchy. It is not a coincidence that its integrable flow on the Universal Grassmann Manifold cut across the Jacobian flows of traditional hierarchies, and this is where the two variables of the sigma function (the Jacobian, and the modular ones) should unite to explain the mystery of the Painleves equations.

2.11. **Algebraic Coding Theory.** Emma's primary contribution to this area is through mentoring undergraduate and graduate thesis or funded-research projects. In fact, this research strand began at the prompting of students in computer science who asked her to give a course on curves over fields of prime characteristic, which she ran for years as a vertically-integrated seminar. Together with her PhD student Drue Coles, she published research papers pursuing Trygve Johnsens innovative idea of error-correction for Goppa codes implemented via vector bundles [73–75], then she pursued overviews and extensions of Goppa codes to surfaces [76].

2.12. **Other.** Emma edited or co-edited four books [77–80]. In addition to book and journal publication, Emma published reviews (BAMS, SIAM), entries in mathematical dictionaries or encyclopaedias, teaching manuals and online research or teaching materials; she also published on the topic of mentoring in the STEAM disciplines.

3. Articles in this volume

Gesztesy and Nichols consider a particular class of integral operators $T_{\gamma,\delta}$ in $L^2(\mathbb{R}^n)$, $n \in \mathbb{N}$, $n \geq 2$, with integral kernels bounded. These integral operators (and their matrix-valued analogs) naturally arise in the study of multi-dimensional Schrödinger and Dirac-type operators and we describe an application to the case of massless Dirac-type operators.

H. Knörrer is using quaternions to give explicit formulas for the global symmetries of the three dimensional Kepler problem. The regularizations of the Kepler problem that are based on the Hopf map and on stereographic projections, respectively, are interpreted in terms of these symmetries.

Zoladek gives two proofs of the Jacobi identity for the Poisson bracket on a symplectic manifold.

Luen-Chau Li presents an expository account of the work done in the last few years in understanding a matrix Lax equation which arises in the study of scalar hyperbolic conservation laws with spectrally negative pure-jump Markov initial data. He begins with its extension to general $N \times N$ matrices, which is Liouville integrable on generic coadjoint orbits of a matrix Lie group. In the probabilistically interesting case in which the Lax operator is the generator of a pure-jump Markov process, the spectral curve is generically a fully reducible nodal curve. In this case, the equation is not Liouville integrable, but we can show that the flow is still conjugate to a straight line motion, and the equation is exactly solvable. En route, we establish a dictionary between an open, dense set of lower triangular generator matrices and algebro-geometric data which plays an important role in the analysis.

F. Calogero and F. Payandeh study solvable dynamical systems in the plane with polynomial interactions. They present a few examples of algebraically solvable dynamical systems characterized by 2 coupled Ordinary Differential Equations. These findings are obtained via a new twist of a recent technique to identify dynamical systems solvable by algebraic operations, themselves explicitly identified as corresponding to the time evolutions of the zeros of polynomials the coefficients of which evolve according to algebraically solvable (systems of) evolution equations.

V. Dragovich, M. Radnovic, R.F. Ranomenjanahary present recent results about double reflection and incircular nets. The building blocks are pencils of quadrics, related billiards and quad graphs.

Alessandro Arsie and Paolo Lorenzoni present a survey of the work done by the authors in the last few years developing the theory of bi-flat F-manifolds and exploring their relationships with integrable hierarchies (dispersionless and dispersive), with Painlevé transcendents, and with complex reflection groups.

Pol Vanhaecke studies some algebraic-geometrical aspects of the periodic 6-particle Kac-van Moerbeke system. This system is known to be algebraically integrable, having the affine part of a hyperelliptic Jacobian of a genus two curve as the generic fiber of its momentum map. Particular attention goes to the divisor needed to complete this fiber into an Abelian variety: it consists of six copies of the curve, intersecting according to a pattern which is determined in the paper. The author also compares this divisor to the divisor which appears in some natural singular compactification of the fiber.

T. Brown and N. Ercolani focus on discrete Painleve equations and connections between combinatorics and integrable systems. Two discrete dynamical systems are discussed and analyzed whose trajectories encode significant explicit information about a number of problems in combinatorial probability, including graphical enumeration on Riemann surfaces and random walks in random environments. The authors show that the two models are integrable and their analysis uncovers the geometric sources of this integrability and uses this to conceptually explain the rigorous existence and structure of elegant closed form expressions for the associated probability distributions. Connections to asymptotic results are also described. The work brings together ideas from a variety of fields including dynamical systems theory, probability theory, classical analogues of quantum spin systems, addition laws on elliptic curves, and links between randomness and symmetry.

T. Kappeler, P. Topalov study the Arnold-Liouville theorem for integrable PDEs. They present an infinite dimensional version of the Arnold-Liouville theorem.

A. Chern, F. Knoeppel, F. Pedit, and U. Pinkall study commuting Hamiltonian flows of curves in real space forms. They provide a geometric point of view of the Hamiltonian flows.

Franco Magri writes on the Kowalewski' s Top from the viewpoint of bihamiltonian geometry. The paper is a commentary of one section of the celebrated paper by Sophie Kowalewski on the motion of a rigid body with a fixed point. Its purpose is to show that the results of Kowalewski may be recovered by using the separability conditions obtained by Tullio Levi Civita in 1904.

Steven Rayan and Jacek Szmigielski study Peakons and Hitchin systems. They review the Calogero-Françoise integrable system, which is a generalization of the

Camassa-Holm system. We express solutions as (twisted) Higgs bundles, in the sense of Hitchin, over the projective line. We use this point of view to (a) establish a general answer to the question of linearization of isospectral flow and (b) demonstrate, in the case of two particles, the dynamical meaning of the theta divisor of the spectral curve in terms of mechanical collisions. They also outline the solution to the inverse problem for CF flows using Stieltjes' continued fractions.

Spalding and Veselov study the the tropical version of Markov dynamics on the Cayley cubic. They prove that this action is semi-conjugated to the standard action of $SL_2(\mathbb{Z})$ on a torus and therefore ergodic with the Lyapunov exponent and entropy given by the logarithm of the spectral radius of the corresponding matrix.

G.S. Mauleshova and A.E.Mironov focus on one-point commuting difference operators. They study a new class of rank one commuting difference operators containing a shift operator with only positive degrees. We obtain equations which are equivalent to the commutativity conditions in the case of hyperelliptic spectral curves. Using these equations we construct explicit examples of operators with polynomial and trigonometric coefficients.

References

[1] Emma Previato, *Gruppi in cui la relazione di Dedekind è transitiva*, Rend. Sem. Mat. Univ. Padova **54** (1975), 215–229 (1976). MR0466319

[2] ———, *Una caratterizzazione dei sottogruppi di Dedekind di un gruppo finito*, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) **59** (1975), no. 6, 643–650 (1976). MR0480738

[3] ———, *Sui sottogruppi di Dedekind nei gruppi infiniti*, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) **60** (1976), no. 4, 388–394. MR0491981

[4] ———, *Groups in whose dual lattice the Dedekind relation is transitive*, Rend. Sem. Mat. Univ. Padova **58** (1977), 287–308 (1978). MR543147

[5] ———, *A lattice-theoretic characterization of finitely generated solvable groups*, Istit. Veneto Sci. Lett. Arti Atti Cl. Sci. Mat. Natur. **136** (1977/78), 7–11. MR548255

[6] ———, *Some families of simple groups whose lattices are complemented*, Boll. Un. Mat. Ital. B (6) **1** (1982), no. 3, 1003–1014. MR683488

[7] R. Freese, *Subgroup lattices of groups by roland schmidt,* 1994.

[8] Emma Previato, *Hyperelliptic quasiperiodic and soliton solutions of the nonlinear Schrödinger equation*, Duke Math. J. **52** (1985), no. 2, 329–377. MR792178

[9] ———, *HYPERELLIPTIC CURVES AND SOLITONS*, ProQuest LLC, Ann Arbor, MI, 1983. Thesis (Ph.D.)–Harvard University. MR2632885

[10] Letterio Gatto and Emma Previato, *A remark on Griffiths' cohomological interpretation of Lax equations: higher-genus case*, Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. **126** (1992), no. 3-4, 63–70. MR1231817

[11] Emma Previato, *Geometry of the modified KdV equation*, Geometric and quantum aspects of integrable systems (Scheveningen, 1992), 1993, pp. 43–65. MR1253760

[12] Emma Previato and Jean-Louis Verdier, *Boussinesq elliptic solitons: the cyclic case*, Proceedings of the Indo-French Conference on Geometry (Bombay, 1989), 1993, pp. 173–185. MR1274502

[13] Bert van Geemen and Emma Previato, *Prym varieties and the Verlinde formula*, Math. Ann. **294** (1992), no. 4, 741–754. MR1190454

[14] ———, *Heisenberg action and Verlinde formulas*, Integrable systems (Luminy, 1991), 1993, pp. 61–80. MR1279817

[15] Christian Pauly and Emma Previato, *Singularities of* 2Θ*-divisors in the Jacobian*, Bull. Soc. Math. France **129** (2001), no. 3, 449–485. MR1881203

[16] E. Previato, T. Shaska, and G. S. Wijesiri, *Thetanulls of cyclic curves of small genus*, Albanian J. Math. **1** (2007), no. 4, 253–270. MR2367218

[17] Emma Previato, *A particle-system model of the sine-Gordon hierarchy*, Phys. D **18** (1986), no. 1-3, 312–314. Solitons and coherent structures (Santa Barbara, Calif., 1985). MR838338

[18] M. R. Adams, J. Harnad, and E. Previato, *Isospectral Hamiltonian flows in finite and infinite dimensions. I. Generalized Moser systems and moment maps into loop algebras*, Comm. Math. Phys. **117** (1988), no. 3, 451–500. MR953833

[19] Emma Previato, *Flows on r-gonal Jacobians*, The legacy of Sonya Kovalevskaya (Cambridge, Mass., and Amherst, Mass., 1985), 1987, pp. 153–180. MR881461

[20] ———, *Generalized Weierstrass $\wp$-functions and KP flows in affine space*, Comment. Math. Helv. **62** (1987), no. 2, 292–310. MR896099

[21] Silvio Greco and Emma Previato, *Spectral curves and ruled surfaces: projective models*, The Curves Seminar at Queen's, Vol. VIII (Kingston, ON, 1990/1991), 1991, pp. Exp. F, 33. MR1143110

[22] Emma Previato and George Wilson, *Vector bundles over curves and solutions of the KP equations*, Theta functions—Bowdoin 1987, Part 1 (Brunswick, ME, 1987), 1989, pp. 553–569. MR1013152

[23] Geoff Latham and Emma Previato, *Higher rank Darboux transformations*, Singular limits of dispersive waves (Lyon, 1991), 1994, pp. 117–134. MR1321199

[24] Geoff A. Latham and Emma Previato, *Darboux transformations for higher-rank Kadomtsev-Petviashvili and Krichever-Novikov equations*, Acta Appl. Math. **39** (1995), no. 1-3, 405–433. KdV '95 (Amsterdam, 1995). MR1329574

[25] ———, *KP solutions generated from KdV by "rank 2" transference*, Phys. D **94** (1996), no. 3, 95–102. MR1392449

[26] E. Previato, *Burchnall-Chaundy bundles*, Algebraic geometry (Catania, 1993/Barcelona, 1994), 1998, pp. 377–383. MR1651105

[27] Emma Previato and George Wilson, *Differential operators and rank 2 bundles over elliptic curves*, Compositio Math. **81** (1992), no. 1, 107–119. MR1145609

[28] Min Ho Lee and Emma Previato, *Grassmannians of higher local fields and multivariable tau functions*, The ubiquitous heat kernel, 2006, pp. 311–319. MR2218024

[29] E. Colombo, G. P. Pirola, and E. Previato, *Density of elliptic solitons*, J. Reine Angew. Math. **451** (1994), 161–169. MR1277298

[30] J. C. Eilbeck, V. Z. Enolskii, and E. Previato, *Varieties of elliptic solitons*, J. Phys. A **34** (2001), no. 11, 2215–2227. Kowalevski Workshop on Mathematical Methods of Regular Dynamics (Leeds, 2000). MR1831289

[31] J. Chris Eilbeck, Victor Z. Enolski, and Emma Previato, *Spectral curves of operators with elliptic coefficients*, SIGMA Symmetry Integrability Geom. Methods Appl. **3** (2007), Paper 045, 17. MR2299846

[32] E. Previato, *Jacobi varieties with several polarizations and PDE's*, Regul. Chaotic Dyn. **10** (2005), no. 4, 531–543. MR2191376

[33] Emma Previato, *The Calogero-Moser-Krichever system and elliptic Boussinesq solitons*, Hamiltonian systems, transformation groups and spectral transform methods (Montreal, PQ, 1989), 1990, pp. 57–67. MR1110372

[34] ———, *Monodromy of Boussinesq elliptic operators*, Acta Appl. Math. **36** (1994), no. 1-2, 49–55. MR1303855

[35] ———, *Reduction theory, elliptic solitons and integrable systems*, The Kowalevski property (Leeds, 2000), 2002, pp. 247–270. MR1916786

[36] Ron Y. Donagi and Emma Previato, *Abelian solitons*, Math. Comput. Simulation **55** (2001), no. 4-6, 407–418. Nonlinear waves: computation and theory (Athens, GA, 1999). MR1821670

[37] È. Previato and V. Z. Ènolʹ skiĭ, *Ultra-elliptic solitons*, Uspekhi Mat. Nauk **62** (2007), no. 4(376), 173–174. MR2358755

[38] Robert D. M. Accola and Emma Previato, *Covers of tori: genus two*, Lett. Math. Phys. **76** (2006), no. 2-3, 135–161. MR2235401

[39] G. N. Benes and E. Previato, *Differential algebra of the Painlevé property*, J. Phys. A **43** (2010), no. 43, 434006, 14. MR2727780

[40] Emma Previato, *Poncelet's theorem in space*, Proc. Amer. Math. Soc. **127** (1999), no. 9, 2547–2556. MR1662198

[41] ———, *Some integrable billiards*, SPT 2002: Symmetry and perturbation theory (Cala Gonone), 2002, pp. 181–195. MR1976669

[42] Yuji Kodama, Shigeki Matsutani, and Emma Previato, *Quasi-periodic and periodic solutions of the Toda lattice via the hyperelliptic sigma function*, Ann. Inst. Fourier (Grenoble) **63** (2013), no. 2, 655–688. MR3112844

[43] Bert van Geemen and Emma Previato, *On the Hitchin system*, Duke Math. J. **85** (1996), no. 3, 659–683. MR1422361
[44] E. Previato, *Dualities on* $\mathcal{T}^*\mathcal{SU}_X(2,\mathcal{O}_X)$, Moduli spaces and vector bundles, 2009, pp. 367–387. MR2537074
[45] W. M. Oxbury, C. Pauly, and E. Previato, *Subvarieties of* $\mathcal{SU}_C(2)$ *and* 2θ*-divisors in the Jacobian*, Trans. Amer. Math. Soc. **350** (1998), no. 9, 3587–3614. MR1467474
[46] Emma Previato, *Theta functions, old and new*, The ubiquitous heat kernel, 2006, pp. 347–367. MR2218026
[47] ———, *Seventy years of spectral curves: 1923–1993*, Integrable systems and quantum groups (Montecatini Terme, 1993), 1996, pp. 419–481. MR1397276
[48] Jean-Luc Brylinski and Emma Previato, *Koszul complexes, differential operators, and the Weil-Tate reciprocity law*, J. Algebra **230** (2000), no. 1, 89–100. MR1774759
[49] Emma Previato, *Another algebraic proof of Weil's reciprocity*, Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei (9) Mat. Appl. **2** (1991), no. 2, 167–171. MR1120136
[50] Alex Kasman and Emma Previato, *Commutative partial differential operators*, Phys. D **152/153** (2001), 66–77. Advances in nonlinear mathematics and science. MR1837898
[51] ———, *Factorization and resultants of partial differential operators*, Math. Comput. Sci. **4** (2010), no. 2-3, 169–184. MR2775986
[52] Maurice J. Dupré, James F. Glazebrook, and Emma Previato, *A Banach algebra version of the Sato Grassmannian and commutative rings of differential operators*, Acta Appl. Math. **92** (2006), no. 3, 241–267. MR2266488
[53] ———, *Curvature of universal bundles of Banach algebras*, Topics in operator theory. Volume 1. Operators, matrices and analytic functions, 2010, pp. 195–222. MR2723277
[54] ———, *Differential algebras with Banach-algebra coefficients I: from* C^**-algebras to the* K*-theory of the spectral curve*, Complex Anal. Oper. Theory **7** (2013), no. 4, 739–763. MR3079828
[55] ———, *Differential algebras with Banach-algebra coefficients II: The operator cross-ratio tau-function and the Schwarzian derivative*, Complex Anal. Oper. Theory **7** (2013), no. 6, 1713–1734. MR3129889
[56] Emma Previato and Mauro Spera, *Isometric embeddings of infinite-dimensional Grassmannians*, Regul. Chaotic Dyn. **16** (2011), no. 3-4, 356–373. MR2810984
[57] Emma Previato, *Multivariable Burchnall-Chaundy theory*, Philos. Trans. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. **366** (2008), no. 1867, 1155–1177. MR2377688
[58] ———, *Lines on abelian varieties*, Probability, geometry and integrable systems, 2008, pp. 321–344. MR2407603
[59] Alexandru Buium and Emma Previato, *Arithmetic Euler top*, J. Number Theory **173** (2017), 37–63. MR3581908
[60] Eleanor Farrington and Emma Previato, *Symbolic computation for Rankin-Cohen differential algebras: a case study*, Math. Comput. Sci. **11** (2017), no. 3-4, 401–415. MR3690055
[61] Jiryo Komeda, Shigeki Matsutani, and Emma Previato, *The sigma function for Weierstrass semigoups* $\langle 3,7,8\rangle$ *and* $\langle 6,13,14,15,16\rangle$, Internat. J. Math. **24** (2013), no. 11, 1350085, 58. MR3143604
[62] ———, *The Riemann constant for a non-symmetric Weierstrass semigroup*, Arch. Math. (Basel) **107** (2016), no. 5, 499–509. MR3562378
[63] J. C. Eilbeck, V. Z. Enolski, S. Matsutani, Y. Ônishi, and E. Previato, *Abelian functions for trigonal curves of genus three*, Int. Math. Res. Not. IMRN **1** (2008), Art. ID rnm 140, 38. MR2417791
[64] ———, *Addition formulae over the Jacobian pre-image of hyperelliptic Wirtinger varieties*, J. Reine Angew. Math. **619** (2008), 37–48. MR2414946
[65] J. C. Eilbeck, V. Z. Enolskii, and E. Previato, *On a generalized Frobenius-Stickelberger addition formula*, Lett. Math. Phys. **63** (2003), no. 1, 5–17. MR1967532
[66] Shigeki Matsutani and Emma Previato, *Jacobi inversion on strata of the Jacobian of the* C_{rs} *curve* $y^r = f(x)$, J. Math. Soc. Japan **60** (2008), no. 4, 1009–1044. MR2467868
[67] ———, *A generalized Kiepert formula for* C_{ab} *curves*, Israel J. Math. **171** (2009), 305–323. MR2520112
[68] ———, *Jacobi inversion on strata of the Jacobian of the* C_{rs} *curve* $y^r = f(x)$*, II*, J. Math. Soc. Japan **66** (2014), no. 2, 647–692. MR3201830

[69] ———, *The al function of a cyclic trigonal curve of genus three*, Collect. Math. **66** (2015), no. 3, 311–349. MR3384012
[70] E. Previato, *Sigma function and dispersionless hierarchies*, XXIX Workshop on Geometric Methods in Physics, 2010, pp. 140–156. MR2767999
[71] Shigeki Matsutani and Emma Previato, *From Euler's elastica to the mKdV hierarchy, through the Faber polynomials*, J. Math. Phys. **57** (2016), no. 8, 081519, 12. MR3541543
[72] ———, *A class of solutions of the dispersionless KP equation*, Phys. Lett. A **373** (2009), no. 34, 3001–3004. MR2559804
[73] Drue Coles and Emma Previato, *Goppa codes and Tschirnhausen modules*, Advances in coding theory and cryptography, 2007, pp. 81–100. MR2440171
[74] ———, *Decoding by rank-2 bundles over plane quartics*, J. Symbolic Comput. **45** (2010), no. 7, 757–772. MR2645976
[75] Emma Previato, *Vector bundles in error-correcting for geometric Goppa codes*, Algebraic aspects of digital communications, 2009, pp. 42–80. MR2605297
[76] Brenda Leticia De La Rosa Navarro, Mustapha Lahyane, and Emma Previato, *Vector bundles with a view toward coding theory*, Algebra for secure and reliable communication modeling, 2015, pp. 159–171. MR3380380
[77] R. Donagi, B. Dubrovin, E. Frenkel, and E. Previato, *Integrable systems and quantum groups*, Lecture Notes in Mathematics, vol. 1620, Springer-Verlag, Berlin; Centro Internazionale Matematico Estivo (C.I.M.E.), Florence, 1996. Lectures given at the First 1993 C.I.M.E. Session held in Montecatini Terme, June 14–22, 1993, Edited by M. Francaviglia and S. Greco, Fondazione CIME/CIME Foundation Subseries. MR1397272
[78] David A. Ellwood and Emma Previato (eds.), *Grassmannians, moduli spaces and vector bundles*, Clay Mathematics Proceedings, vol. 14, American Mathematical Society, Providence, RI; Clay Mathematics Institute, Cambridge, MA, 2011. Papers from the Clay Mathematics Institute (CMI) Workshop on Moduli Spaces of Vector Bundles, with a View Towards Coherent Sheaves held in Cambridge, MA, October 6–11, 2006. MR2809924
[79] Emma Previato (ed.), *Advances in algebraic geometry motivated by physics*, Contemporary Mathematics, vol. 276, American Mathematical Society, Providence, RI, 2001. MR1837106
[80] ——— (ed.), *Dictionary of applied math for engineers and scientists*, Comprehensive Dictionary of Mathematics, CRC Press, Boca Raton, FL, 2003. MR1966695
[81] ———, *Soliton equations and their algebro-geometric solutions. Vol. I [book review of mr1992536]*, Bull. Amer. Math. Soc. (N.S.) **45** (2008), no. 3, 459–467. MR3077138
[82] E. Previato, *Poncelet's porism and projective fibrations*, Higher genus curves in mathematical physics and arithmetic geometry, 2018, pp. 157–169. MR3782465
[83] ———, *Complex algebraic geometry applied to integrable dynamics: concrete examples and open problems*, Geometric methods in physics XXXV, 2018, pp. 269–280. MR3803645
[84] Alexandru Buium and Emma Previato, *The Euler top and canonical lifts*, J. Number Theory **190** (2018), 156–168. MR3805451
[85] E. Previato, *Curves in isomonodromy and isospectral deformations: Painlevé VI as a case study*, Algebraic curves and their applications, 2019, pp. 247–265. MR3916744
[86] Jiryo Komeda, Shigeki Matsutani, and Emma Previato, *The sigma function for trigonal cyclic curves*, Lett. Math. Phys. **109** (2019), no. 2, 423–447. MR3917350

Mathematics Department, UPenn, Philadelphia, PA, 19104-6395

Department of Mathematics and Statistics, Oakland University, Rochester, MI. 48309